Reprinted from JOURNAL OF APPLIED PHYSICS, Vol. 40, No. 9, 3771–3775, August 1969

Steady Shock Profile in a One-Dimensional Lattice

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(Received 6 March 1969; in final form 28 April 1969)

The equations of motion of a one-dimensional lattice of mass points connected by nonlinear springs are set forth and compared with the equations of the corresponding continuum. A permanent regime for the damped lattice is obtained by series approximation and shown to agree with that of the continuum. A higher approximation leads to a permanent regime profile for the undamped lattice which oscillates steadily after shock arrival. This is shown to be in qualitative accord with the results of numerical integrations of the transient problem. However, comparison of periods of steady oscillation with those obtained in the transient problem indicate that the series approximation to the permanent regime is quantitatively unsatisfactory, though qualitatively correct. Scaling of the problem with a parameter $u_1\alpha$ is noted, where u_1 is steady particle velocity behind the shock and α is a parameter of nonlinearity.

I. INTRODUCTION

Considerable attention has been given to the discussion of steady shock-compression profiles in gases.[1] Much less work has been done on the analogous problem in solids, partly because a satisfactory microscopic model of a solid is not available, partly because the mathematics of nonlinear lattices is more formidable than that of random atomic assemblies, and partly because interest in shock waves in solids has generally tended to lag behind that in gases. Band[2] has discussed in general terms the steady profile problem. Bland[3] has obtained explicit profiles in a continuum for particular assumptions about the constitutive relations. These efforts are based principally upon continuum models of solids and require, as in gases, existence of time-dependent forces for definition of shock profiles.

When lattice models of solids are being considered, the processes for introducing dissipative mechanisms are less straightforward than for a continuum, since dissipation is now to be described in terms of irreversible relative motions of atoms which form the lattice or of their constituents. Anderson[4] has obtained steady profiles in a one-dimensional lattice with nonlinear forces by introducing dashpots in parallel with springs connecting atoms. It is shown in Sec. III of this paper that such a model leads to a smooth, non-oscillatory shock transition between two uniform states and that the transition is the analogue of that which occurs in the continuum, provided a certain expansion is properly truncated.

Numerical solutions of transient shock wave problems in lattices without dissipation have shown that even in such cases the shock profile has finite rise time and is oscillatory but not steady. The amplitude of oscillations behind the shock front decays with the passage of time

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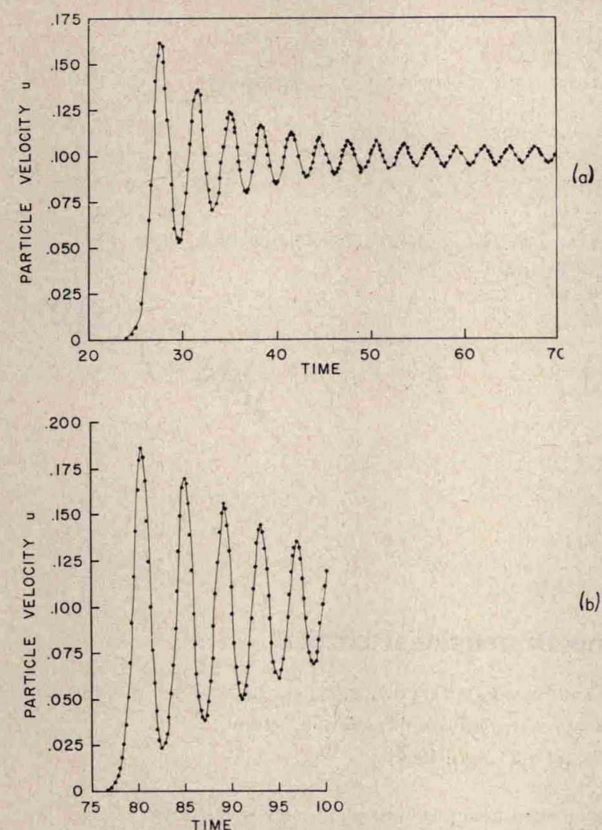

FIG. 1. Transient shock profiles from numerical integrations for semi-infinite lattice driven by step change in velocity: (a) 30 particles from driven end, (b) 90 particles from driven end.

because the lattice is dispersive.[5-7] Some typical results of such numerical integrations are shown in Fig. 1. Profiles of the kind shown there are disturbing for two reasons: (i) they are not steady, and all our experience in the continuum, which should be a limit of the lattice, indicates that steady profiles do exist, and (ii) the one nonlinear lattice problem which can be solved exactly, viz. the sliding of perfectly elastic beads on a wire, as in Fig. 2, has each particle oscillating indefinitely with constant amplitude after the shock wave has passed. Such behavior constitutes, in the present context, a steady profile. A detailed examination of the more general problem of a one-dimensional lattice with nearest neighbor interaction and without dissipation is undertaken in Sec. IV. The mathematical problem posed is unusual, but an approximate permanent regime solution is found which is in harmony with the results shown in Figs. 1 and 2, though some disagreements between this solution and the transient case still exist.

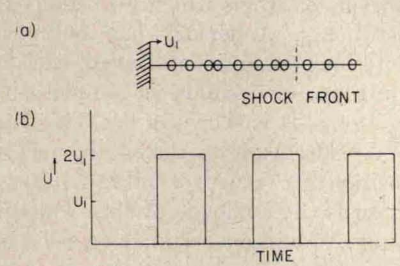

FIG. 2. Velocity profile for shock in a system of beads sliding on a wire: (a) bead positions and shock front at a particular time, (b) permanent regime profile for each bead.

II. EQUATIONS OF MOTION

The lattice model is illustrated in Fig. 3, including dissipative dashpots, as introduced by Anderson. The entire lattice is generated by translation of a single mass–spring–dashpot element, and mass points are constrained to move in the direction of the lattice. The separation between undisturbed masses is x_0. The sign convention used in describing forces is shown in Fig. 3. It is chosen opposite from that normally used because these forces will be compared with pressures, not stresses,

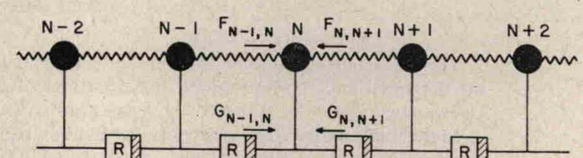

FIG. 3. Lattice model with damping.

in the continuum case. With this convention the force, $F_{N,N+1}$, exerted on mass N by $N+1$ is negative when the spring connecting N and $N+1$ is stretched beyond its equilibrium position. We assume the force to be nonlinear with parabolic form:

$$F_{N,N+1}=-(S_{N+1}-S_N)+\alpha(S_{N+1}-S_N)^2. \qquad (1)$$

Dimensionless variables are used here and in the equations following. The relative velocity of the two particles is assumed to generate a linear damping force:

$$G_{N,N+1}=-\eta(S_{N+1}'-S_N'), \qquad (2)$$

where $'\equiv d/dT$ and T is a dimensionless time. Combining Eqs. (1) and (2) with similar forces due to motion of the $N-1$ particle leads to an equation of motion:

$$S_N''(T)=-(F_{N,N+1}-F_{N-1,N})-(G_{N,N+1}-G_{N-1,N}). \qquad (3)$$

In order to pass to the continuum limit for uniaxial strain, we suppose that space is filled with parallel lattices like the one shown, one per unit area, and that $x_0=N\Delta x_0$ is a Lagrangian coordinate for the Nth

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